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Experimental evidence for a power law in electroencephalographic α–wave dynamics

Y. Georgelin[1], L. Poupard[2], R. Sartène[2], and J.C. Wallet[1,a]

[1] Groupe de Physique Théorique, Institut de Physique Nucléaire, 91406 Orsay Cedex, France
[2] Laboratoire d'Explorations Fonctionnelles, Hôpital Robert Ballanger, 93602 Aulnay-sous-bois, France

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Abstract. We perform an experimental study of the time behavior of the α-wave events occurring in human electroencephologhraphic signals. When the subjects have to keep their eyes open, we find that the fraction of the time spent in an α-burst of time size τ exhibits a scaling behavior as a function of τ. The corresponding exponent is equal to 1.75 ± 0.13. Furthermore, we show that our experimental result may have a possible explanation within a class of Self-Organized Critical (SOC) models recently proposed by Boettcher and Paczuski. In particular, one of these models, when properly re-interpreted, appears to be consistent both with our experimental result and a physiological description of the possible origin of α-wave events.

PACS. 87.10.+e General theory and mathematical aspects – 05.45.-a Nonlinear dynamics and nonlinear dynamical systems

1 Introduction

Trying to understand and encode in rather simple models the fundamental properties underlying the richness and complexity of biological systems and functions has become a major topics in modern biology. Despite their apparent complexity, some of these systems/functions exhibit, among other features, a tendency for organization as well as self-organization which can occur at various levels. A simple example in morphogenesis is provided by the organized variability observed in the branching structure of the lung which can be explained by scaling arguments, first introduced a long time ago in [1] and further developed in [2]. The concept of scaling is now well established in biology and physiology (for a review see *e.g.* [3]) and appears to be a useful tool to understand features of many processes. In particular, scaling shows up in the power law behavior of some observables.

The human brain is one of the most complex physiological systems. It involves billions of interacting physiological and chemical processes giving rise to the experimentally observed neuroelectrical activity. The corresponding dynamics exhibits a complicated behavior which reflects itself in electrophysiological recordings, namely the electroencephalographic recordings (EEG), which, roughly speaking, are assumed to capture the mean electrical activity generated by the ($\mathcal{O}(10^7)$) neurons involved in the small area of the cortex (says, $\mathcal{O}(1)\text{cm}^2$) surrounding the electrode. Recall that the cortex is the outer most (2 mm thick) layer of the brain. The attempts to extract relevant information from the neuroelectrical activity have generated a large amount of investigations for more than 20 years, the former one mainly based on the Fourier analysis of the time series stemming from the EEG signal [4]. In order to quantify the EEG, models for the origin of the EEG that take into account some of the anatomical and neurophysiological features of the brain were developed [5], in particular continuum models offering a way to describe the macroscopic scale (wave-like) electrical activity [6] of the EEG and numerical simulations both at macroscopic and microscopic (cellular) scale[1] [7]. Besides, the use of powerful methods inherited from non linear physics have provided a deeper insight into the fundamental properties ruling the observed neuroelectrical dynamics and, in particular, the possible occurrence of self-organization in the cortical electrical activity has been suggested in recent works [8], but so far no evidence for scaling laws in the corresponding dynamics has been reported.

One of the major difficulties to observe a power law in human neuroelectrical activity is to determine relevant observables from the EEG signal. Recall that the evolution from a deep sleep to an (active) awakening level reflects itself into four dominant regimes of the EEG signal which are conventionally classified according to their frequency range [9]. These four regimes are

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[a] e-mail: `wallet@ipno.in2p3.fr`

[1] together with studies on the complementarity of the various models and attempting to bridge the microscopic and macroscopic scales

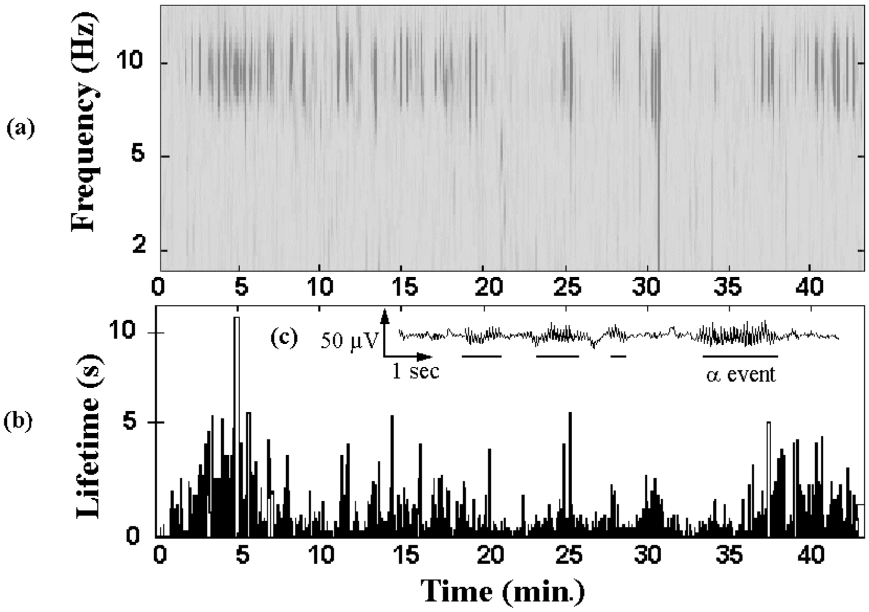

Fig. 1. On panel (a) is depicted the time average of the square modulus of the wavelet transform of a typical EEG signal whose maxima, indicated by the darkest areas, correspond to α-events. The corresponding lifetimes are collected on panel (b). An example of successive α-events occurring in the EEG signal is presented in panel (c).

called δ-waves ([0.5 Hz, 4 Hz]), θ-waves ([4 Hz, 8 Hz]), α-waves ([8 Hz, 12 Hz]) and β-waves ([13 Hz, 19 Hz]) (the lowest frequency range δ corresponding to a deep sleep level). It is known that α-waves occur when human awakening level drops down slowly towards sleep [9]. α-waves represent therefore an electroencephalographic landmark of drowsiness. Successive α-wave events/bursts can be observed for a rather long period (up to a few hours) with typical time size (lifetime) from $\mathcal{O}(100)$ms up to $\mathcal{O}(10)$s. They can be easily isolated from the background EEG activity so that they are good candidates for study. Figure 1 shows successive α-wave events with different lifetimes. Notice the irregular variations of the occurrence times between the onset of two successive α events.

In this paper, we study the lifetime of α-wave events occurring in EEG signals recorded on two groups of human subjects having a mild sleep deprivation. In the first group, all the subjects have to keep their eyes open whereas in the second one they keep their eyes closed. The signal processing is performed using a standard wavelet transform analysis [10] which appears to be well-suited to deal with the transients involved in the EEG's and in particular to extract reliably the various α-wave events [11]. For each EEG signal, we measure the cumulated time for α-events with fixed time size τ, normalized to the total duration of the EEG signal (which basically represents the fraction of the time spent in an α-burst of time size τ), hereafter denoted by $P_{\rm exp}(\tau)$. For the first group of subjects (eyes open), we find that $P_{\rm exp}(\tau)$ has a power law form given by $P_{\rm exp}(\tau) \sim \tau^{-\omega}$ with $\omega = 1.75 \pm 0.13$. This provides a new example of a power law with fractional exponent appearing in this area of physiology. Furthermore, we show that this experimental result may have a possible explanation within a class of Self-Organized Critical (SOC) models recently discussed in the physics literature [12]. In particular, one of these models, when properly re-interpreted, appears to be consistent both with our experimental result and a physiological description of the possible origin of α-wave events, therefore suggesting that this model may be of some relevance for the description of the time distribution of the *transitions* between α and "non α" states, that is, the temporal architecture of the α-bursts.

2 A power law in the α-wave dynamics

Let us first describe briefly the pure experimental part of this work (*i.e.* the data recording). The experimental procedure consists in recording the EEG activity of two groups of subjects (defined below) who all have had a four hours sleep deprivation during the previous night. Each subject was installed in the sitting posture for a two-hour EEG recording. Each EEG signal was obtained from temporal and occipital electrode location and was further filtered through a [0.5 Hz, 30 Hz]-bandpass and digitally converted at a rate of 200 samples/s. In order to test the influence of the visual cortex and other neural visual pathways on the development of α-waves, two groups of subjects have been considered: the group 1 (resp. 2) involving 10 (resp. 5) subjects having to keep their eyes open (resp. closed) corresponding to visual (resp. non visual) relaxed thinking.

To get more insight into the dynamics governing the occurrence of α-bursts, we choose the α-events lifetime as a representative physical observable [13]. The various α-events (and corresponding lifetimes) are easily extracted from any EEG signal $s(t)$ using standard wavelet analysis [10,11]. In particular, α-events correspond to those part of the signal whose wavelet transform modulus is maximum in the α-frequency range [8 Hz, 12 Hz]. Recall that the continuous one-dimensional wavelet transform is given by [10]

$$(W_{\psi s})(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{+\infty} \mathrm{d}t \; s(t) \; \psi^* \left(\frac{t-b}{a} \right), \qquad (1)$$

where the real parameters a ($a > 0$) and b are respectively the scale and time parameter, $\psi(t)$ is the so-called mother function and * denotes complex conjugation. In what follows, we choose $\psi(x) = \pi^{1/4} \exp(\mathrm{i}\theta_0 x) \exp(-x^2/2)$ where θ_0 is a numerical constant [14], which is particularly suitable for frequency characterization and offers a good compromise between frequency resolution and time localization [11]. In the numerical analysis, we consider the discrete version of (1) which can be written as

$$(W_{\psi s})(n, a) = \left(\frac{\delta t}{a}\right)^{1/2} \sum_{n'=0}^{N-1} s(n')\psi^* \left(\frac{(n'-n)\delta t}{a}\right) \quad (2)$$

for any EEG signal $s(n\delta t)$ (n integer) of total duration $N\delta t$, where δt is the time step.

It is convenient to consider the time average of the square modulus of (2). The corresponding expression is given by

$$\langle |(W_{\psi s})(m\delta t', a)|^2 \rangle = \sum_{n=mk}^{(m+1)k-1} \frac{1}{k} |(W_{\psi s})(n\delta t, a)|^2,$$
$$k = \frac{\delta t'}{\delta t};\ m = 0, 1, ..., \left(\frac{N}{k} - 1\right) \quad (3)$$

where k is a reduction factor from δt to $\delta t'$ [15], this later being identified with the uncertainty in time localization. This permits one to disregard the events whose time duration is shorter than $\delta t'$ (and also to eliminate spurious effects due to EEG background noise). Then, any α-burst will correspond to the part of the signal for which (3) is maximum when the scale parameter a belongs to a range associated with the α-frequency range [8 Hz, 12 Hz]. The corresponding lifetime can then be straightforwardly obtained from (3).

We have extracted all the α-events from the EEG activity in each of the EEG signals and determined the corresponding lifetimes. This allows us to define $P_{\exp}(\tau)$, the fraction of the time spent in an α-burst of time size τ. For the group 1 (subjects with eyes open), we find that this quantity exhibits a scaling behaviour, $P_{\exp}(\tau) \sim \tau^{-\omega}$. The corresponding exponent is found to be

$$\omega = 1.75 \pm 0.13, \quad (4)$$

where the second term in (4) (standard deviation) reflects both the inter individual variability and artifacts such as eyes motions and/or muscular activity (which are inherent to EEG measurement). A similar analysis performed for the data obtained from the group 2 (subjects with eyes closed) leads to the conclusion that $P_{\exp}(\tau)$ does not obey a power law. Indeed, a careful inspection of the data shows clearly that no slope can be defined when plotting $\log(P_{\exp}(\tau))$ *versus* $\log(\tau)$ (Fig. 2).

The comparison of both experimental results first indicates that there is likely to be an important influence of the visual cortex on the development of α-waves when the eyes are open. Moreover in this later situation, the fact that the overall temporal pattern of α-waves (and in particular the temporal behaviour of the *transitions* between, says, an α-state and a "non α" one) exhibits a power law may reflect a specific property of the corresponding dynamics. It is therefore interesting to examine what may be the origin of the observed scaling law in the distribution of the time size of periods of α activity which characterizes the dynamics of the transitions between α and "non α"-states. Developing a complete model of EEG[2] reproducing the above experimental observations is beyond the scope of the present paper. In what follows, we will try to analyze if the observed power law can be recovered in the framework of a statistical model that includes some features of a commonly accepted physiological description, owing to the fact that one has to deal with a system having a large number of degrees of freedom, intermittency in the α activity whose temporal dynamics is characterized by the scaling exponent (4). To do this, we adopt a phenomenological viewpoint.

3 Discussion

Let us first start with general physiological considerations. Although α-wave occurrence is an important feature of the EEG activity, the corresponding precise cellular mechanisms are not well known. However, it is commonly accepted that α-waves have a cortical origin and are driven by presynaptic inputs from the thalamic level with local cortical factors [16]. Notice that the measurement of the α-waves through the EEG cannot be viewed as a measurement representing a global phenomenon of the cortex. Indeed, it is known that the observed α-activity depends strongly of the considered area of the cortex. Furthermore, even on an area where the α-activity can occur significantly at a given time, noticeable spatial variations in the behavior of the α-bursts can be currently observed for different positions of the electrode on this area.

When drowsiness occurs, a sleep inducing mechanism may alter some global and/or local control parameters of small clusters of neurons (*e.g.* $\mathcal{O}(10^3 - 10^4)$ neurons). Typically, these clusters may represent corticocortical columns with excitatory and inhibitory neurons which could initiate α-waves in the EEG by the way of multiple-length scale interactions present in the cortex [5]. Corticocortical columns are small functional units with no specific afferents that provide multiple excitatory and inhibitory inputs to other (*e.g.* $\mathcal{O}(10 - 100)$) corticocortical columns. Note that the electrode of the EEG captures the electrical activity generated typically by $\mathcal{O}(10^3 - 10^4)$ clusters (or equivalently by $\mathcal{O}(10^7)$ neurons as announced in the introduction).

We are now in position to propose a simple tentative interpretation of the power law that we have observed experimentally in the α-wave dynamics within the framework of a class of statistical models. To do this, we first notice that the temporal architecture of

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[2] Notice that most of the EEG models are devoted to the study of wavelike processes corresponding to steady states of the cerebral cortex whereas in the present situation one has to deal with transitions between states.

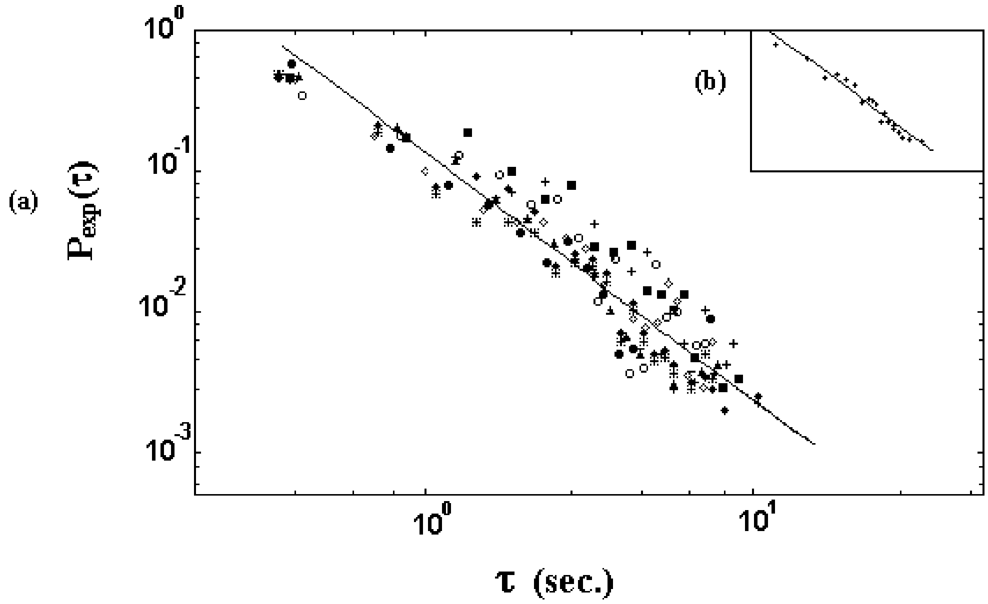

Fig. 2. Log-log plot of the fraction of time spent in an α-burst of lifetime τ *versus* τ. All the data for the subjects are collected in Figure 2a. The straight line depicted in Figure 2a corresponds to an exponent equal 1.75, obtained by first fitting the data for each subject by using the mean square method (see Fig. 2b for a typical example for a given subject) and then averaging the results over the 10 subjects. The corresponding standard deviation is equal to 0.13.

the α-bursts which is currently observed, where basically peculiar changes are concentrated in time intervals interrupting periods of inactivity, is somehow similar to a punctuated equilibrium behavior. This later appears in particular within a class of SOC models [12], called multi-trait models, which can be viewed as extensions of the original Bak-Sneppen model [17]. These models are defined as follows (for more details see [12] and Ref. therein): each site of a D-dimensional lattice is labelled by M numbers belonging to the unit interval. At every time step, the smallest number in the lattice is replaced by a new number randomly chosen from a flat distribution in the unit interval, whereas one of the M numbers on each neighboring site is also randomly replaced by a new random number taken from the flat distribution.

In order to exhibit a possible link between the above mentioned models and the present experimental situation, we introduce the following set of hypothesis: First, we assume that the relevant part of the cortex that give rise to the α activity that we have observed here can be modeled by an effective 1-dimensional lattice. Then, each site is identified with a cluster of neurons (that is, a cortico-cortical column). Since each cluster is in fact under the control of a large number of parameters (external neuronal inputs, neuromodulators, ion channels, etc.), it is reasonable to consider the limit $M \to \infty$. Furthermore, we assume that an α-burst may start when some control parameters driving the activity level of only one cluster among the $\mathcal{O}(10^3 - 10^4)$ clusters monitored by the electrode of the EEG drop below some threshold. In this situation, this cluster would become incapable of information processing, (therefore becoming presumably isolated from the other ones). The α-wave would then presumably last until information can be processed again, due to some reactivation mechanism. In other words, the α activity detected by the electrode would therefore correspond to the inactivation of only one column among the $\mathcal{O}(10^3 - 10^4)$ ones that are covered by the electrode.

Before going further, some comments are in order. It appears that the last hypothesis, which basically means that only one (or a few) specific unit/center is responsible for the signal detected by the electrode, underlies a noticeable part of the studies on the EEG aiming to interpret observational facts in the framework of self-organization. For an experimental neurophysiological discussion, see [18], and references therein. Now, according to the previous discussion, and assuming that the above hypothesis are valid, the $D = 1$, $M \to \infty$ multi-trait model appears naturally to be suitable for providing a possible interpretation of the experimentally observed power law. This model has been considered in detail in [12] and is known to represent a different universality class than the Bak-Sneppen model. Its punctuated equilibrium behavior has been characterized in particular through $P_F(\tau)$ the distribution of (time) sizes of periods of inactivity (~isolation) for a given site. In turn, this can then be identified with $P_{\rm exp}(\tau)$. This quantity, by definition, is simply the distribution of time periods of isolation of some cortical area. The distribution $P_F(\tau)$ has been shown [12] to obey a power law given by

$$P_F(\tau) \sim \tau^{-7/4}, \tag{5}$$

whose exponent is in good agreement with the one given in (4) which characterizes the scaling behavior of $P_{\rm exp}(\tau)$ that we have determined experimentally. This is consistent with the $D = 1$ $M \to \infty$ multi-trait model being of some relevance to the dynamics of the α-bursts in EEG activity[3].

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[3] The corresponding exponent in the $D = 2$ directed percolation model is equal to 1.84 [19]. Strictly speaking, this model is still consistent with our experimental result although the existence of a preferred direction in the cortex is difficult to reconcile with the present physiological knowledge. We therefore consider this model as rather unsuitable for describing the α-wave dynamics.

Let us summarize the main result of this paper. We have first identified experimentally a new power law occurring in the temporal pattern of α-waves which is observed whenever the eyes are open. The influence of the visual cortex has been also examined experimentally and appears to be important for the occurrence of the power law. We then try to propose a tentative interpretation of the above result in the framework of a particular (statistical) model in order to examine if the power law might be the indirect signature of a specific property ruling the dynamics of the α-burts. We have found that our experimental result may be understood in the framework of a particular SOC model, namely the $D = 1$ $M \to \infty$ multitrait model [12], provided some hypothesis are valid. We note en passant that the dynamics of the α-rhythms would then appear to give a hint toward a theoretical proposal regarding self-organization of cerebral activity [20].

Since our analysis suggests that the above mentioned SOC model may be successfully applied to describe some of the dynamics of the α-bursts for which, consequently, self-organization and punctuated equilibrium behavior may well play a salient role, some experimental investigation on the validity of the hypothesis underlying the present analysis must be performed. In particular, it is important to test experimentally the existence of a specific corticocortical column responsible for the detected α activity among those covered by the electrode together with the nature of the reactivation mechanism for the columns which should result from the interconnection of the columns. Obviously, this cannot be directly done on human because it would need to perform electrophysiological recordings at the level of one or a few clusters of neurons directly in the cortex, to localize in particular possible candidates responsible for the α-activity (*i.e.* α-generators). However, a possible way to overcome this difficulty would be to carry out such experimental investigations on animals (*e.g.* mamalians) for which a somehow comparable situation in the EEG would have been identified, that is, for which a specific EEG activity mimicking the human α-activity is known to exist and which would exhibit a power law in the corresponding temporal pattern for some given environmental condition.

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14. In the present numerical analysis, $\theta_0 = 5.5$. For a discussion on the Morlet-Grossmann wavelet that we choose, see reference [10]; see also D. Gabor, J. IEE **93**, 429 (1946).
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